Mathematical Giftedness in the Brain

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ABSTRACT

Humans, other mammals, and birds, all have a well-evolved sense of the numerosity of small sets of objects. However, our brains did not evolve to do mathematics. Instead, our brains learn mathematics at school by coordinating neural interconnectivity. Doing mathematics critically involves:

- the lateral frontal cortices to support working memory;
- the temporal cortices (and hippocampus) to reconstruct knowledge from long term memory;
- the orbitofrontal cortices and the anterior cingulate for decision making, in turn mediated by regions within the limbic sub-cortex;
- areas of the fusiform gyri and temporal lobes for sequencing of symbolic representations;
- the parietal lobes for spatial reasoning about conceptual inter-relationships;
- the cerebellum for mental rehearsal.

Neuroimaging studies of mathematically gifted subjects show enhanced bilateral participation of these contributing brain areas: the brains of gifted children have enhanced interconnectivity. Neuroimaging performance at arithmetic, algebra, geometry, statistics, and calculus, shows that the parietal lobe is crucial for all

mathematics. As a main role of the parietal lobe is spatial processing, its involvement in mathematical thinking seems consistent with reports from school teachers that abilities at spatial reasoning are good predictors of gifted children's prowess at mathematics.

ARITHMETIC IN THE BRAIN

Neuroimaging studies reveal at least ten separate areas of the cortex, from the rear to the front, and across both sides, that contribute to the simple task of subtracting one number from another. In an early functional magnetic resonance imaging (fMRI) study, French cognitive neuroscientist Stanislas Dehaene (1997) put himself in the scanner to investigate which parts of his brain were involved in repeated subtraction: 100 take away 7, take away 7, take away 7, and so on. Areas of increased activation included the left and right fusiform gyri (imagining the numbers), left and right parietal cortices (number sense), lateral and medial parts of the temporal lobe (arithmetic memories), and inferior parts of the frontal lobes (working memory, and decision making). A process as simple as subtraction requires a network of interconnected specialist brain functional modules.

There is also good evidence for differences between networks of modules for each of the four basic arithmetic operations, addition, subtraction, multiplication, and division, and a separate network for estimation (bilateral inferior parietal cortex) as opposed to computation (left parietal and frontal cortices). This is not to say that these networks are completely independent of one another. They all have parts in common, notably frontal and parietal areas, but they also include areas of brain which seem to be unique to each arithmetical operation. In other words, there is no specific brain area or module for doing arithmetic. Rather, arithmetical brain functioning seems to rely on the co-operation of discrete modules located in many different parts of the brain across both hemispheres.

As well as unique networks for the four major arithmetic operations, different brain systems are involved in solving problems presented orally or in a written format, whether the problems require number comprehension or number production, and whether the task demands reading or writing numbers vs. calculation (Dowker, 2005). Could preferred arithmetic computational strategies reflect individual differences in the strengths of neural connectivities, e.g. doing multi-column addition from the left, or doing mental subtraction by rounding off before taking away?

One of the fascinating procedural dissociations is between processing small numbers 1 to 6 (literally a handful) and larger numbers 7, 8, 9, and zero. This seems best explained by evolution. Look at the number of objects on the table or desk in front of you. If there are five or fewer, you can tell this at a glance – you don't need to count. But if there are more, say, a dozen, then you will need to count, or at least to break them into a couple of smaller groups. Young infants, even a few months old, seem to be able to distinguish between 1, 2, 3, 4 and sometimes even 5 objects (Dowker, 2005). This is not to say that infants have a sense of absolute number magnitude. Presumably they do not. But it is to suggest that they do have some 'inbuilt' sense of small number in at least a relative manner. Perhaps preschool and reception/ kindergarten classes could assume this basic knowledge in their pre-numeracy curriculum.

And not only do humans have an 'inbuilt' number sense up to around five; so do most mammals, including apes, monkeys, dolphins, dogs, cats, rats, and horses, as well as birds, including parrots, pigeons and crows (Dehaene, 1997; Dowker, 2005). One could conjecture that this sense of a handful might have evolved under the adaptive pressure of keeping track of one's children, or immediate relatives. A different suggestion is that such a number sense enables an animal in a group, when faced with a group of prey, to compute the odds of success at either standing and fighting, or fleeing. Of course, these speculative accounts are not incompatible. And interestingly, chimpanzees have been taught to recognise numbers of objects up to nine, and to order them by relative magnitude (Boysen & Berntson, 1986). What is important is that a strong 'inbuilt' number sense can be both a cognitive strength on which to build an early number curriculum, and a cognitive weakness in that it could bias number processing away from less intuitive constructs. Early indications of giftedness in pre-school classes can be gleaned through open assessment of children's number sense, especially an early sense of decimal organisation.

ALGEBRA IN THE BRAIN

Such a recommendation assumes that school learning requires neural system building: strengthening neural connections between disparate functional modules throughout the brain. The sorts of changes that we as teachers want to induce in the brains of our students should resemble those of adults who are competent, even expert, in the subject area in question. One neuroimaging study investigated the change of the brain activation patterns as children learned algebra equation solving (Qin et al, 2004). The participants were ten pre-algebra adolescents who were tutored for five days in linear algebraic equations. These were presented in three levels of difficulty in terms of the number of steps required to reach a solution, e.g., 0-step: $1x + 0 = 4$; 1-step: $1x + 8 = 12$; 2-step: $7x + 1 = 29$. Pleasingly, there was a significant reduction in the solving time required over the five days.

During this period, their patterns of brain activations while solving similar algebraic equations were compared with the activation patterns of adult algebra 'experts' solving the same equations. The areas of significant activation were very similar, and included the inferior parietal cortex, the prefrontal cortex, and the anterior cingulate. But what was interesting was that there was a significant reduction in the activation in the parietal and prefrontal cortices for the adolescents over the five days, consistent with the widely held neuroscientific model of synaptic reinforcement as a necessary neural process for learning by children of new subject matter.

GEOMETRY IN THE BRAIN

Of course these algebraic equations were quite trivial as equations go, and being successful at school mathematics, much less beyond, requires a far greater facility at complex reasoning. To better understand how this process differs between experts, proficient problem solvers, and novices, an fMRI study investigated complex reasoning in Euclidean geometric proof (Kao & Anderson, 2006). Here 15 young adults attempted to provide proofs for the equality of length for pairs of sides in triangular figures. The knowledge required was the Euclidean properties of triangles. The experimental design not only varied the degree of difficulty (and impossibility), but had the relevant sides highlighted in colour in half the figures.

The results showed a beneficial interaction effect for the colour highlighting, suggesting that proficient problem-solvers integrate problem givens and diagram information to support their logical inferences. The areas of the brain which were the most responsive were the left parietal and right prefrontal cortices, the same areas involved in arithmetic and algebraic problem solving. The implication for teaching geometric proof is that highlighting the objects of the proof in the diagram might help maintain focused attention, especially for students with less expansive working memory capacities.

GIFTED MATHEMATICAL BRAIN FUNCTION

There have been several neuroimaging studies of the brain function of mathematically gifted children compared with normal age-matched peers. O'Boyle et al (2005) used fMRI to study the brain functioning of mathematically able boys performing number sentence completion and spatial rotation tasks, i.e., imagining how a group of blocks might appear once they've been turned around. This is the sort of thinking involved in doing 3-dimensional geometry, whereas number sentence completion involves pre-algebraic thinking. Consistent with previous EEG studies of gifted adolescents (O'Boyle, 2000), this fMRI study found that, among other areas, the right parietal and frontal areas of the brain were involved in both pre-algebraic <u>and</u> geometrical thinking of able young mathematicians.

But, whereas previous neuroimaging studies had shown that mental block rotation tasks induce only parietal activity on the right, these gifted subjects demonstrated bilateral activation of the parietal lobes and frontal cortex, along with heightened activation of the anterior cingulate, during mental rotation. The researchers conjectured that:

> *[I]t may be that enhanced (and bilateral) activation of the parietal lobes, frontal cortex, and the anterior cingulate are critical parts of an all-purpose information processing network, one that is relied upon by individuals who are intellectually gifted, irrespective of the nature of their exceptional abilities.* (O'Boyle et al, 2005, p. 586)

Further evidence for enhanced bilaterality as a neural characteristic of mathematical giftedness includes the results from psychophysical studies aimed to elicit laterality biases in gifted subjects (Singh & O'Boyle, 2004). In sum:

> *[E]nhanced development and subsequent processing reliance on the specialized capacities of the right hemisphere, coupled with a fine-tuned ability for rapid and coordinated exchange of information between the hemispheres, are hypothesized to be unique processing characteristics of the mathematically gifted brain.* (Singh & O'Boyle, 2004, p. 676)

In other words, mathematical thinking requires the coordinated participation of several neural systems, which in the brains of gifted mathematicians seem more extensive throughout both right and left hemispheres. These neural systems involve at least the temporal cortices for the storage and retrieval of number facts and computational rules and algorithms, the parietal cortices for number sense and conceptual interrelationships, especially of a quasi-spatial representations, the anterior cingulate, a region involved in emotionally-weighted decision making, and the frontal cortices for working memory, creative analogising, and cognitive coordination of the other on-frontal systems.

This could explain the high levels of arithmetic competence of young gifted children, who typically show a fascination for number patterns and relations, e.g., factors, which feature conceptual connectivity. They are exploiting their characteristic robust neural interconnectivity, especially between the hemispheres (Singh & O'Boyle, 2004).

Another feature of the brain functioning of mathematically gifted children is their enhanced frontal cortical activation when engaged in mathematical problem solving. Evidence for the importance of frontal functioning for mathematical giftedness comes from an early PET study where the brain activations of a mathematically gifted group of college students were significantly different from their age-matched peers in their prefrontal cortices, prompting the researchers to conclude that the prefrontal cortices are critically involved in mediating high mathematical intelligence (Haier & Benbow, 1995). Many neuroimaging studies since have highlighted the central role of the frontal cortices in cognitive demands of high intelligence (e.g., Strange et al, 2001; Kroger et al, 2002).

Neuroimaging studies led by John Duncan at the University of Cambridge have shown that frontal areas on both sides of the brain are involved in a great range of general high intelligence tasks, including reasoning, memorisation, and linguistic expression (Duncan et al, 2000). Duncan accounts for the multi-functioning of frontal brain areas with an adaptive model of neuronal coding (Duncan, 2001). Neurons in these areas of the frontal cortices are able to adapt their information processing to the demands of the task at hand, be it recreation of data from long-term memory, or logical inference, or information categorisation, or monitoring progress of the current task, and so on. Moreover, adaptive frontal functioning maintains task commitment through persistent activation of relevant inputs from other brain areas. This model can explain the importance of frontal functioning in creative mathematical thinking: here is a region which can compare and contrast putative ideas to generate candidate solutions to the mathematical problem at hand (Geake & Dobson, 2005) through greater efficacy of working memory with gifted individuals (Geake, 2008a). Or, since these processes are largely unconscious, produce candidate solutions after a good night's sleep.

SPATIAL REASONING IN MATHEMATICS

The frontal cortex, however, is not the only brain region which contributes to the integration of information processing, especially with high cognitive challenge. The parietal cortex is especially involved in spatial perception and spatial working memory (Jung & Haier, 2007). For example, an EEG study of the brain functioning of gifted Korean adolescents by Lee et al (2006) concluded that:

> *superior-g may not be due to the recruitment of additional brain regions but to the functional facilitation of the fronto-parietal network particularly driven by the posterior parietal activation.* (Lee et al, 2006, p. 578)

Parietal functioning contributes to mathematical giftedness in two complementary ways. An American fMRI study of spatial imagery in deductive reasoning found that reasoning activated an occipito-parietal–frontal network, including the prefrontal cortex, the cingulate gyrus, the superior and inferior parietal cortex, and the visual association cortex (Knauff et al, 2002). This network of brain systems enables reasoners to envisage and inspect spatially organized mental models to solve deductive inference problems. So, one specific contribution of the parietal cortex seems to be as a mental 'draughtsman' for spatial, and quasi-spatial, perceptions.

But as we've seen above, the parietal cortex also acts as a substrate for a number sense. This has been well-demonstrated, for example, in an event-related fMRI study where Eger et al (2003) presented numbers, letters, and colours in the visual and auditory modality, asking subjects to respond to target items within each category. In the absence of explicit magnitude processing, numbers compared with letters and colours across modalities activated a bilateral region in the horizontal intra-parietal sulcus. The researchers concluded that this intra-parietal response supports the idea of number representation which is independent of presentation modality, and which is automatically triggered by the presentation of numbers to code magnitude information.

This dual role raises an evolutionary question: Why does the parietal cortex compute magnitude <u>and</u> create mental visualisation? The answer is that the parietal cortex enables us, and our evolutionary ancestors, to find our way home. Both sorts of information processing are necessary to find our way in the world. A mental map of where we are going seems obvious, but without a sense of magnitude of the distances we have travelled, and orientations we have turned, our mental map reading is likely to go astray. In the real world we are helped, of course, by remembering landmarks. This integrative process of navigation has been the subject of French research, which found that during locomotion, mammals update their position with respect to a fixed point of reference, such as their point of departure, by processing inertial cues, proprioceptive feedback and stored motor commands generated during locomotion (Etienne, Maurer & Séguinot, 1996).

This so-called path integration system (dead reckoning) allows the animal to return to its home, or to a familiar feeding place, even when external cues are absent or novel. However, without the use of external cues, the path integration process leads to rapid accumulation of errors involving both the direction and distance of the goal. Both path integration and familiar visual cues cooperate to optimise navigational performance. The dual roles of the parietal cortex seem to have evolved to help us find our way by engaging whole-body propriocentrism to perform geometry and trigonometry in the real world.

This leads directly to a highly recommendable classroom application: LOGO Turtle! When Seymour Papert (1993) introduced LOGO he was at pains to point out that this was not simply an exercise in computer programming, but a cognitive task requiring children to step out the geometric shape required in order to devise the LOGO commands for the turtle on the basis of the child's propriocentric experience. For example, a square is so many steps in a straight line, then a right or left hand turn through 90 degrees, then the same number of steps, then another 90 degree turn in the same orientation, and repeat until back at the starting point. The LOGO program, with some careful thought, can reduce to an algorithm of FWD and RT x 4, although this is not the only possible solution. The point is that LOGO might well be particularly useful in helping students 'see' mathematical problems, and their solutions.

THE 'AHA' EXPERIENCE

A notable behavioural characteristic of gifted mathematicians is their enjoyment of solving a mathematical problem, the emotional satisfaction arising from the 'Aha!' experience. The neural activity behind such a subjective feeling has been investigated with fMRI, to locate the regions within the brain especially involved, and with EEG to record the temporal dynamics (Jung-Beeman et al, 2004). The stimuli were problems which could be solved with insight vs. those whose solutions were obvious or non-insightful. FMRI revealed increased activity in the right temporal area during initial solving efforts, and this activity remained when the solution was insightful compared with a non-insightful solution. EEG recordings revealed a sudden burst of high-frequency neural activity in the same area beginning 0.3 s prior to an insightful solution. Given that this right temporal area is associated with making connections across distantly related information during comprehension, it seems that the sudden flash of insight occurs when these distinct neural systems are suddenly coupled, enabling cognitive processes that allow perception of conceptual connections that had previously not been apparent.

To get to the ‘aha’ end-point, gifted mathematical thinking involves a high degree of creativity. This in turn requires analogical thinking in a fluid manner, a cognitive characteristic of gifted individuals (Geake, 2008b). A good example in recent times is Andrew Wiles’ solution of Fermat’s Last Theorem, involving establishing a hitherto unproven relationship between modular functions and elliptical groups (Singh, 2006). The lesson from Wiles’s work is that progress in one area of mathematics can be achieved through making creative or fluid analogies with another.

There is neuroimaging evidence that the frontal cortices support analogical and related types of reasoning, whether simple analogies (e.g., black is to white as high is to … ?) (Luo et al., 2003), or spatial analogies (Wharton et al, 2000), and including deductive reasoning (Prabhakaran et al, 1997), and reasoning underpinning relational complexity (Christoff et al, 2001; Kroger et al, 2002). In sum, the results of all of these studies converge on the suggestion that creative mathematical thinking, just like computational processing, is reliant on a network involving the parietal and frontal areas of the brain. This network is distinct from another involving the temporal and frontal areas which seems to support computation, the main aspect of academic or school mathematics.

The suggestion that mathematical thinking requires contributions from both academic and creative abilities might explain some popularised extremes in mathematical performance. First there are the autobiographical reports of a number of great mathematicians, such as Benoit Mandelbrot of fractals fame, who performed relatively poorly at primary school arithmetic, where opportunities for creative input were largely absent. In contrast, it is certainly the case that calculation savants (as portrayed in the film *Rain Man*) cannot do mathematics in the constructive and creative manner that professional mathematicians do. It could be hypothesised that in some cases of mathematical genius, ordinary performance at primary school arithmetic, and even poor adult performance at adding up the shopping bill, might be indicative of a relatively under-developed academic ability, at least in the specific task of number computation, whereas the case of the autistic-savant who can calculate calendar week days, or square roots of large numbers, the creative side of mathematics might be impaired. But this is not to say that extremes in these two aspects of mathematical ability are always incompatible. Far from it. There are many professional mathematicians who are "calculating geniuses". Perhaps the best know is Richard Feynman, Nobel Laureate in theoretical physics, whose biography (Gleick, 1992) provides an insightful account of how such gifted mathematicians employ their intellectual skills to reorganise calculation strategies based on their deep numerical knowledge.

Based on my own neuroimaging research into the neural bases of fluid analogising (Geake & Hansen, 2005; in progress), I would argue that a necessary prerequisite for creative thinking is knowledge, relevant and sometimes apparently irrelevant. Similarly, creative mathematical thinking requires extensive and thorough mathematical knowledge (Geake, 2006). Outstanding mathematical talent can best be nurtured, then, even in an intrinsically motivated student with high task commitment, through engagement in challenging mathematical activities, requiring an in-depth knowledge of mathematics (Bacon et al, 1991).

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